

Generalized Wigner-Inönü Contractions and Maxwell (Super)Algebras

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We consider a class of generalized Inönü-Wigner contraction for semidirect product of two particularly related semisimple Lie (super)algebras. The special class of such contractions provides D=4 Maxwell algebra and recently introduced simple D=4 Maxwell superalgebra. Further we present two types of D=4 N -extended Maxwell superalgebras, the nonstandard one for any N with $\frac{1}{2}N(N-1)$ central charges and the standard one, for even $N=2k$, with $k(2k-1)$ internal symmetry generators.

I. INTRODUCTION

Standard Wigner-Inönü contraction (IW) [1, 2] one applies usually to the symmetric cosets of simple Lie algebras, i.e. $\hat{g} = \hat{h} \oplus \hat{k}$, where

$$[\hat{h}, \hat{h}] \subset \hat{h} \quad [\hat{h}, \hat{k}] \subset \hat{k} \quad [\hat{k}, \hat{k}] \subset \hat{h}. \quad (1.1)$$

After the following rescaling of the generators $h_\alpha \in \hat{h}$ and $k_i \in \hat{k}$

$$h_\alpha = h'_\alpha \quad k_i = \lambda k'_i, \quad (1.2)$$

one obtains in the limit $\lambda \rightarrow \infty$ the contracted algebra \hat{g}' which becomes a semidirect product

$$\hat{g}' = \hat{h} \ltimes \hat{k}', \quad (1.3)$$

with Abelian sector \hat{k}' . In such a way one can obtain¹ e.g. the Poincaré algebra $O(3,1) \ltimes T^4$ from anti-de-Sitter ($\hat{g} = O(3,2)$) or de-Sitter ($\hat{g} = O(4,1)$) algebras, as well as the simple superPoincaré algebra $\tilde{\mathcal{P}}_4 = O(3,1) \ltimes (T^4 \oplus Q^4)$ (Q^4 describe four real odd supercharges) by choosing $\hat{g} = OSp(1|4)$ ($N=1$, D=4 *AdS* superalgebra) and $\hat{h} = O(3,1)$ (D=4 Lorentz algebra). It should be mentioned that the standard W-I contraction procedure has been extended as well to quantum - deformed Lie algebras [3, 4] and provided first in the literature quantum Poincaré algebra [5, 6].

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¹ In this note for simplicity we shall consider only the case D=4.

The standard W-I contraction scheme can be extended in various ways to the non simple Lie algebras (see e.g. [7]). The general scheme which we shall use below was proposed by Weimar-Woods [8] and discussed further in [9].²

Let us assume that the nonsimple Lie algebra \widehat{G} is decomposed into the $n+1$ sets of generators $\widehat{g}^{(k)}$ ($k = 0, 1, \dots, n$)

$$\widehat{G} = \widehat{g}^{(0)} \oplus \widehat{g}^{(1)} \oplus \dots \oplus \widehat{g}^{(n)}, \quad (1.4)$$

where the following conditions are satisfied [8]

$$[\widehat{g}^{(k)}, \widehat{g}^{(l)}] \subset \bigoplus_{s \leq k+l} \widehat{g}^{(s)} \quad (1.5)$$

The contraction of (1.4) is obtained by properly rescaling the generators

$$\widehat{g}^{(i)} = \lambda^{a_i} \widehat{g}^{(i)}, \quad (1.6)$$

with the choice of a_i providing the finite limits of the algebra (1.5) if $\lambda \rightarrow \infty$.

Clearly it follows that

- i) $\widehat{g}^{(0)}$ describes the subalgebra of \widehat{G}
- ii) if $n=1$ we obtain standard decomposition of Lie algebra into a Riemannian pair with $\widehat{g}^{(1)}$ describing a coset \widehat{k} (see (1.1))

In this paper we shall consider the contraction of the following semidirect product which provides a particular choice of (1.4–1.5) for $n=2$ ³

$$\widehat{G} = \widehat{h} \ltimes \widehat{g} = \widehat{h} \ltimes (\widehat{h} \oplus \widehat{k}). \quad (1.7)$$

We see that the algebra \widehat{h} occurs twice and all algebraic relations in algebra (1.7) are already given by (1.1). One can rewrite (1.7) using rescaled generators.

$$\widehat{G} = \widehat{h} \ltimes (\widehat{h}' \oplus \widehat{k}''). \quad (1.8)$$

where

$$h_\alpha = \lambda^2 h'_\alpha \quad k_i = \lambda k''_i. \quad (1.9)$$

² We shall call it the WI W-W (Wigner, Inönü, Weimar-Woods) contraction scheme.

³ The extension of presented scheme to superalgebras \widehat{G} is straightforward.

In the limit $\lambda \rightarrow \infty$ we obtain the following set of commutation relations

$$\begin{aligned} [\widehat{h}, \widehat{h}] &\subset \widehat{h} & [\widehat{h}, \widehat{h}'] &\subset \widehat{h}' & [\widehat{h}, \widehat{k}''] &\subset k'' \\ [\widehat{k}'', \widehat{k}''] &\subset \widehat{h}' & [\widehat{h}', \widehat{k}''] &= 0 & [\widehat{h}', \widehat{h}'] &= 0. \end{aligned} \quad (1.10)$$

We see that the generator k''_i are the “Lie-algebraic roots” of the Abelian generators h'_α .⁴

In this note we shall apply such a contraction scheme to the derivation in Sect. 2 of Maxwell algebra [10]–[12] and simple nonstandard [13] as well as standard [14] Maxwell superalgebras. In Sect. 3 by performing suitable WI W-W contractions we shall obtain new results: the extended nonstandard and extended standard Maxwell superalgebras.

In Sect. 4 we shall recall briefly the geometric interpretation of Maxwell (super)symmetries [14, 15] and present final comments.

II. MAXWELL ALGEBRA AND SIMPLE MAXWELL SUPERALGEBRAS BY WI W-W CONTRACTION SCHEME

i) Maxwell algebra (arbitrary D)

Let us choose in the formula (1.7)

$$\widehat{h} = O(D-1, 1) \quad \widehat{g} = O(D-1, 2). \quad (2.1)$$

We denote the generators \widehat{h} of Lorentz algebra by $M_{\mu\nu}$, and the generators \widehat{g} of D -dimensional AdS algebra by $(\widetilde{M}_{\mu\nu}, \mathcal{P}_\mu)$ (we put AdS radius $R = 1$). One gets

$$\begin{aligned} [M_{\mu\nu}, M_{\rho\tau}] &= i(\eta_{\nu[\rho} M_{\mu]\tau} - \eta_{\mu[\rho} M_{\nu]\tau}) \\ [M_{\mu\nu}, \widetilde{M}_{\rho\tau}] &= i(\eta_{\nu[\rho} \widetilde{M}_{\mu]\tau} - \eta_{\mu[\rho} \widetilde{M}_{\nu]\tau}) \\ [\widetilde{M}_{\mu\nu}, \widetilde{M}_{\rho\tau}] &= i(\eta_{\nu[\rho} \widetilde{M}_{\mu]\tau} - \eta_{\mu[\rho} \widetilde{M}_{\nu]\tau}) \\ [M_{\mu\nu}, \mathcal{P}_\rho] &= i(\eta_{\nu\rho} \mathcal{P}_\nu - \eta_{\mu\rho} \mathcal{P}_\mu) \\ [\widetilde{M}_{\mu\nu}, \mathcal{P}_\rho] &= i(\eta_{\nu\rho} \mathcal{P}_\mu - \eta_{\mu\rho} \mathcal{P}_\nu) \\ [\mathcal{P}_\mu, \mathcal{P}_\nu] &= i\widetilde{M}_{\mu\nu} \end{aligned} \quad (2.2)$$

The rescaling ($M_{\mu\nu}$ remain unchanged)

$$\widetilde{M}_{\mu\nu} = \lambda^2 Z_{\mu\nu} \quad \mathcal{P}_\mu = \lambda P_\mu \quad (2.3)$$

⁴ This terminology is used in analogy with the statement that supercharges in semisimple Lie superalgebras are called the “SUSY roots” of its bosonic generators.

provides in the limit $\lambda \rightarrow \infty$ the Maxwell algebra:

$$[M_{\mu\nu}, M_{\rho\tau}] = i(\eta_{\nu[\rho} M_{\mu]\tau} - \eta_{\mu[\rho} M_{\nu]\tau}) \quad (2.4a)$$

$$[M_{\mu\nu}, Z_{\rho\tau}] = i(\eta_{\nu[\rho} Z_{\mu]\tau} - \eta_{\mu[\rho} Z_{\nu]\tau}) \quad (2.4b)$$

$$[M_{\mu\nu}, P_\rho] = i(\eta_{\nu\rho} P_\mu - \eta_{\mu\rho} P_\nu) \quad (2.4c)$$

$$[P_\mu, P_\nu] = i Z_{\mu\nu} \quad (2.4d)$$

$$[Z_{\mu\nu}, Z_{\rho\tau}] = [P_\mu, Z_{\rho,\tau}] = 0. \quad (2.4e)$$

We see from (2.4b,2.4d,2.4e) that the Abelian generators $Z_{\rho\tau}$ can be called tensorial central charges⁵.

We add that one can obtain the Maxwell algebra as well if we choose in (2.1) that $\hat{g} = O(D, 1)$ (D -dimensional de-Sitter) algebra [16].

ii) Simple nonstandard D=4 Maxwell superalgebra

The structure presented by formulae (1.7–1.10) can be obtained as well if \hat{g} in (1.7) describes the semisimple Lie superalgebra [8]. We choose in (1.7) for D=4

$$\hat{h} = O(3, 1) \quad \hat{g} = OSp(1|4). \quad (2.5)$$

The $OSp(1|4)$ relations extend the bosonic sector $Sp(4; R) = O(3, 2) = (\widetilde{M}_{\mu\nu}, \mathcal{P}_\mu)$ by four real supercharges \widetilde{Q}_A ($A = 1 \dots 4$) as follows:⁶

$$\begin{aligned} \{\widetilde{Q}_A, \widetilde{Q}_B\} &= \frac{1}{2}(C \gamma^\mu)_{AB} \mathcal{P}_\mu + \frac{1}{2}(C \sigma^{\mu\nu})_{AB} \widetilde{M}_{\mu\nu} \\ [\widetilde{M}_{\mu\nu}, \widetilde{Q}_A] &= \frac{1}{2}(\sigma_{\mu\nu})_{AB} \widetilde{Q}^B \\ [\mathcal{P}_\mu, \widetilde{Q}_A] &= \frac{1}{2}(\gamma_\mu)_{AB} \widetilde{Q}^B \end{aligned} \quad (2.6a)$$

Besides we have the relations

$$[M_{\mu\nu}, \widetilde{Q}_A] = \frac{1}{2}(\sigma_{\mu\nu})_{AB} \widetilde{Q}^B \quad (2.6b)$$

where $M_{\mu\nu} \in \hat{h}$ and $\widetilde{M}_{\mu\nu} \in \hat{g}$ (see (2.5)).

If we keep the rescaling (2.3) of bosonic (even) generators the only rescaling of odd generators leading in the limit $\lambda \rightarrow \infty$ to a nontrivial anticommutator of supercharges is provided by

$$\widetilde{Q}_A = \lambda Q_A. \quad (2.7)$$

⁵ This may explain why Maxwell algebra was rediscovered in [17] under the name of tensorial extension of Poincaré algebra.

⁶ We use real Majorana realization of D=4 Dirac algebra, with $C = \gamma_0$ ($C^T = -C$, $C^2 = -1$).

In such a case we obtain as a contraction limit $\lambda \rightarrow \infty$

$$\{Q_A, Q_B\} = \frac{1}{2} C(\sigma^{\mu\nu})_{AB} Z_{\mu\nu} \quad (2.8)$$

and

$$[Q_A, Z_{\mu\nu}] = [Q_A, \mathcal{P}_\mu] = 0 \quad (2.9)$$

Such supersymmetrization of Maxwell algebra was introduced by Soroka and Soroka [13, 17]. In such nonstandard supersymmetrization scheme only the tensorial central charges are supersymmetrized, and the fourmomentum generators, including energy, are not expressed as bilinear expressions in terms of supercharges Q_A .

Concluding, the N=1 *AdS* superalgebra can provide by contraction only the nonstandard supersymmetrization of Maxwell algebra. In such a framework, contrary to the scheme of SUSY quantum mechanics the Hamiltonian is not expressed in terms of supercharges.

iii) Standard Maxwell superalgebra by WI W-W contraction of $OSp(2|4)$.

Let us write down the basic relation of N=2 *AdS* superalgebra ($r, s = 1, 2$)

$$\{\tilde{Q}_A^r, \tilde{Q}_B^s\} = \frac{1}{2} \delta^{rs} [(C \sigma^\mu)_{AB} \mathcal{P}_\mu + (C \sigma^{\mu\nu})_{AB} \tilde{M}_{\mu\nu} + \epsilon^{rs} C_{AB} Z], \quad (2.10)$$

where internal $O(2)$ symmetry generator Z rotates a pair of multiplets of supercharges

$$[\tilde{Q}_A^r, Z] = \epsilon^{rs} \tilde{Q}_A^s. \quad (2.11)$$

In order to supersymmetrize after contraction both generators P_μ and $Z_{\mu\nu}$ we shall rewrite the superalgebra (2.9) in terms of suitably projected supercharges

$$\tilde{Q}_A^{r(\pm)} \equiv \tilde{Q}_A^r \pm \epsilon^{rs} (\gamma_5)_{AB} \tilde{Q}_B^s. \quad (2.12)$$

One gets the following basic superalgebraic relations

$$\begin{aligned} \{\tilde{Q}_A^{r(\pm)}, \tilde{Q}_B^{s(\pm)}\} &= \frac{1}{2} P_{AC}^{(+rs)} (C \sigma^{\mu\nu})_{CB} \mathcal{P}_\mu \\ \{\tilde{Q}_A^{r(+)}, \tilde{Q}_B^{s(-)}\} &= \frac{1}{2} P_{AC}^{(+rs)} (C \sigma^{\mu\nu})_{CB} M_{\mu\nu} \\ &+ P_{AC}^{(+rt)} \epsilon^{ts} C_{CB} \cdot Z. \end{aligned} \quad (2.13)$$

By choosing besides (2.3) the rescaling [19]

$$\begin{aligned} \tilde{Q}_A^{r(+)' } &= \lambda^{1/2} Q_A^{r(+)} \\ \tilde{Q}_A^{r(-)' } &= \lambda^{3/2} Q_A^{r(-)} \end{aligned} \quad (2.14)$$

and additionally

$$Z = \lambda^2 B_5 \quad (2.15)$$

one gets the following $\lambda \rightarrow \infty$ contraction limit of the relations (2.12)

$$\begin{aligned} \{Q_A^{r(+)}, Q_B^{s(-)}\} &= \frac{1}{2} P_{AC}^{(\pm)rs} (C \gamma^\mu)_{CB} \mathcal{P}_\mu \\ \{Q_A^{r(+)}, Q_B^{s(-)}\} &= \frac{1}{2} P_{AC}^{(+rs)} (C \sigma^{\mu\nu})_{CB} Z_{\mu\nu} \\ &+ P_{AC}^{(+rt)} \epsilon^{ts} C_{CB} B_5. \end{aligned} \quad (2.16)$$

In such a way we obtain the standard Maxwell superalgebra centrally extended by the Abelian generator B_5 as obtained recently in [14, 19]⁷. It should be added that in [14] the projected components of supercharges were expressed as two-component complex Weyl spinors, and the technique of projected supercharges was not used.

III. THE EXTENDED D=4 MAXWELL SUPERALGEBRAS

The N -extended AdS superalgebra $OSp(4|N)$ is given by the following basic relation satisfied by N multiplets of supercharges Q_A^r ($r = 1 \dots N$)

$$\{\tilde{Q}_A^r, \tilde{Q}_B^s\} = \frac{1}{2} \delta^{rs} [(C \gamma^\mu)_{AB} \mathcal{P}_\mu + (C \sigma^{\mu\nu})_{AB} \tilde{M}_{\mu\nu}] + \tilde{T}^{rs} C_{AB}, \quad (3.1)$$

where $\tilde{T}^{rs} = -\tilde{T}^{sr}$ describe $\frac{N(N-1)}{2}$ internal $O(N)$ symmetry generators

$$\begin{aligned} [\tilde{T}^{rs}, \tilde{T}^{tu}] &= \delta^{rt} \tilde{T}^{su} - \delta^{st} \tilde{T}^{ru} \\ &+ \delta^{ru} \tilde{T}^{st} + \delta^{su} \tilde{T}^{rt}. \end{aligned} \quad (3.2)$$

They act on supercharges \tilde{Q}_A^r as follows

$$[\tilde{T}^{rs}, \tilde{Q}_A^t] = \tau^{rs;tu} \tilde{Q}_A^u, \quad (3.3)$$

where $\tau^{rs;tu}$ describe $N \times N$ matrix realization of the algebra (3.2), i.e. provide the fundamental vectorial realization of the orthogonal rotations generators \tilde{T}^{rs} .

a) Nonstandard N -extended D=4 Maxwell superalgebras.

⁷ For complete set of superalgebraic relations, extending odd-odd sector (2.16) by odd-even and even-even ones see [14, 19].

In such a case we supplement the rescalings (2.3) providing the Maxwell algebra with the following relations

$$\tilde{Q}_A^r = \lambda Q_A^r \quad \tilde{T}^{rs} = \lambda^2 \tilde{Z}^{rs}. \quad (3.4)$$

Using (2.3) and (3.4) we obtain the N -extended nonstandard D=4 Maxwell superalgebras

$$\begin{aligned} \{Q_A^r, A_B^s\} &= 2\delta^{rs}(C\sigma^{\mu\nu})_{AB}Z_{\mu\nu} + \tilde{Z}^{rs}C_{AB} \\ [M_{\mu\nu}, Q_A^r] &= \frac{1}{2}(\sigma_{\mu\nu})_{AB}Q_B^r \\ [M_{\mu\nu}, Z_{\rho\tau}] &= i(\eta_{\nu[\rho}Z_{\mu]\tau} - \eta_{\mu[\rho}Z_{\nu]\tau}) \\ [\tilde{Z}^{rs}, \tilde{Z}^{tu}] &= [M_{\mu\nu}, \tilde{Z}^{rs}] = 0 \\ [P_\mu, Q_A^r] &= [Z_{\mu\nu}, Q_A^r] = [\tilde{Z}^{rs}, Q_A^t] = 0. \end{aligned} \quad (3.5)$$

We see that the generators \tilde{Z}^{rs} in (3.5) describe the set of $\frac{N(N-1)}{2}$ scalar central charges. In order to obtain \tilde{Z}^{rs} as the tensorial central charges which transform under internal $O(N)$ as 2-tensors, one should consider the contractions (2.3) and (3.4) applied to the following choice of semidirect product (1.7)

$$\tilde{G} = (O(3,1) \oplus O(N)) \ltimes OSp(N|4) \quad (3.6)$$

b) Standard k -extended Maxwell superalgebra.

Let us consider the contraction of the superalgebra (3.1) for even N ($N = 2k$). We introduce the following projected supercharges (see also [18])

$$\tilde{Q}_A^{(\pm)r} = P_{(k)AB}^{(\pm)rs} \tilde{Q}_B^s, \quad (3.7)$$

where $(r, s = 1, 2, \dots, 2k; \Omega^{rs} = -\Omega^{sr}, \Omega^2 = -1)$

$$P_{(k)AB}^{(\pm)rs} = \frac{1}{2}(\delta_{AB}\delta^{rs} \pm C_{AB}\Omega^{rs}). \quad (3.8)$$

It follows from (3.8) that

$$P_{AB}^{(\pm)rs} P_{BC}^{(\pm)st} = P_{AC}^{(\pm)rt} \quad P_{AB}^{(+rs)} P_{BC}^{(-st)} = 0. \quad (3.9)$$

We choose further

$$\Omega = \begin{pmatrix} 0 & 1_k \\ -1_k & 0 \end{pmatrix}. \quad (3.10)$$

The $2k \times 2k$ antisymmetric operator - valued matrix \tilde{T}^{rs} describing $O(2k)$ generators can be divided into two sectors $\tilde{T}_+^{rs}, \tilde{T}_-^{rs}$ satisfying the relations

$$\tilde{T}_\pm \Omega = \mp \Omega \tilde{T}_\pm^T = \pm \Omega \tilde{T}_\pm. \quad (3.11)$$

We get explicitly ($A = -A^T, C = -C^T, B$ arbitrary)

$$\tilde{T}_\pm = \begin{pmatrix} A & B \\ -B^T & C \end{pmatrix} \quad \begin{aligned} B &= \pm B^T \\ A &= \pm C. \end{aligned} \quad (3.12)$$

From (3.12) follows that the generators \tilde{T}_+^{rs} describe the algebra $U(k)$ and the generators \tilde{T}_-^{rs} the coset $\frac{O(2N)}{U(k)}$.

Using the projected supercharges (3.7) one can rewrite the $OSp(N|4)$ superalgebra as follows:

$$\begin{aligned} \{\tilde{Q}_A^{(\pm)r}, \tilde{Q}_B^{(\pm)s}\} &= \frac{1}{2} P_{AB}^{(\pm)rs} (C \gamma^\mu)_{AB} \mathcal{P}_\mu + C_{AB} \tilde{T}_-^{rs} \\ \{\tilde{Q}_A^{(+r)}, \tilde{Q}_B^{(-s)}\} &= \frac{1}{2} P_{AB}^{(+rs)} (C \gamma^{\mu\nu})_{AB} \tilde{M}_{\mu\nu} + C_{AB} \tilde{T}_+^{rs}. \end{aligned} \quad (3.13)$$

We supplement (2.3) with the following set of rescalings

$$\begin{aligned} \tilde{Q}_A^{(+r)} &= \lambda^{1/2} Q_A^{(+r)} & \tilde{T}_+^{ts} &= \lambda^2 T_+^{rs} \\ \tilde{Q}_A^{(-r)} &= \lambda^{3/2} Q_A^{(-r)} & \tilde{T}_-^{rs} &= \lambda T_-^{rs}. \end{aligned} \quad (3.14)$$

We obtain in the limit $\lambda \rightarrow \infty$ the following contracted superalgebra

$$\begin{aligned} \{\tilde{Q}_A^{(+r)}, \tilde{Q}_B^{(+s)}\} &= 2P_{AB}^{(+rs)} (C \gamma^\mu)_{AB} P_\mu + C_{AB} T_-^{rs} \\ \{\tilde{Q}_A^{(+r)}, \tilde{Q}_B^{(-s)}\} &= 2P_{AB}^{(-rs)} (C \sigma_{\mu\nu})_{AB} Z_{\mu\nu} + C_{AB} T_+^{rs} \\ \{\tilde{Q}_A^{(-r)}, \tilde{Q}_B^{(-s)}\} &= 0. \end{aligned} \quad (3.15)$$

In such a way we get new k -extended standard D=4 Maxwell superalgebra with the internal sector T described by the sum of two Abelian subalgebras T_+^{rs} (k^2 generators) and T_-^{rs} ($k(k-1)$ generators) satisfying the following relations

$$[\hat{T}_-, \hat{T}_-] \subset \hat{T}_+ \quad [\hat{T}_\pm, \hat{T}_\pm] = 0. \quad (3.16)$$

We see therefore that the internal symmetry sector $T = (T_+, T_-)$ has similar algebraic structure as the space-time sector described by Maxwell algebra $(Z_{\mu\nu}, P_\mu)$ (compare (3.16) with (2.4d–2.4e)). If we put $k = 1$ we obtain simple standard Maxwell superalgebra (see Sect. 2.iii), with single internal symmetry generator $T_+ = B_5$ (see (2.15–2.16)).

IV. CONCLUDING REMARKS

We would like to add the following brief comments:

a) The choice (1.7) of nonsimple algebra in the form of semidirect product leads to simple homogeneous rescaling rules (1.6). If we introduce the generators $\hat{j}^{(\pm)} = \hat{h}^{(1)} \pm \hat{h}^{(2)}$, where $\hat{h}^{(i)}$ ($i = 1, 2$) is the pair of commuting \hat{h} -algebras, the algebra \hat{G} can be rewritten equivalently as the direct sum of algebras $\hat{j}^{(+)}$ and $(\hat{j}^{(-)} \oplus \hat{k})$

$$\hat{G} = \hat{j}^{(+)} \oplus (\hat{j}^{(-)} \oplus \hat{k}). \quad (4.1)$$

Such a way of describing the deformed (super)Maxwell algebra which provides after contraction the (super)Maxwell algebras was preferred in [16, 19]⁸.

ii) Maxwell symmetries can be interpreted as describing the symmetries of Minkowski space-time filled uniformly with constant EM field. In such a geometric framework we assume that arbitrary constant values of EM field strength describe new degrees of freedom, which extend the standard relativistic space-time [10–12], [15, 16]. Recently proposed simple standard Maxwell superalgebra describes the supersymmetries in superspace enlarged by arbitrary constant components of the $U(1)$ gauge field strength superfield (for details see [14]).

iii) Addition of new generators to D=4 Poincaré algebra leads naturally to extended space-time geometry. If we wish however to stay e.g. with the framework of D=4 field theory invariant under the (super) Maxwell symmetries, one should realize these new symmetries on the target space of field values, in analogy with the component formulation of the supersymmetric extension of standard D=4 relativistic field theory. These ideas are now under consideration.

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⁸ See also [20] where however the contractions of $O(3, 1) \oplus OSp(N; 4)$ to the standard Maxwell superalgebras were not considered.

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